ON SUPER-EXPONENTIAL ALGORITHM, CONSTANT MODULUS ALGORITHM AND INVERSE FILTER CRITERIA FOR BLIND EQUALIZATION

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ABSTRACT

Super-exponential algorithm (SEA), constant modulus algorithm (CMA) and inverse filter criteria (IFC) using higher-order statistics have been widely used for blind equalization. Chi, Feng and Chen have reported that SEA and IFC are equivalent under certain conditions. In this paper, we further prove that SEA, IFC and CMA are equivalent under certain conditions, and their convergence speed and computational load can be significantly improved as the given data are preprocessed by the well-known lattice linear prediction error (LPE) filter for both off-line processing and adaptive processing. Some simulation results are presented to support the analytic results and the proposed off-line and adaptive implementations.

1. INTRODUCTION

Blind equalization (deconvolution) is a signal processing procedure to recover the desired independent identically distributed (i.i.d.) non-Guassian signal, denoted by $u[n]$, that is transmitted through an unknown linear time-invariant (LTI) channel, denoted by $h[n]$, with only measurements

$$\begin{aligned} x[n] &= u[n] * h[n] + w[n] \\ &= \sum_{k=-\infty}^{\infty} h[k]u[n-k] + w[n] \end{aligned} \tag{1}$$

where $w[n]$ is additive noise. The problem of blind equalization arises comprehensively in a variety of applications such as digital communications, seismic deconvolution, speech modeling and synthesis, ultrasonic nondestructive evaluation and image restoration.

The FIR linear equalizer of order L, denoted by $v[n]$, has been widely used to process $x[n]$ such that the equalizer output

$$\begin{aligned} e[n] &= x[n] * v[n] = \sum_{k=0}^{L} v[k]x[n-k] \\ &= u[n] * g[n] + w[n] * v[n] \quad \text{(by (1))} \end{aligned} \tag{2}$$

approximates $\alpha u[n-\tau]$ ($\alpha \neq 0$) where

$$g[n] = h[n] * v[n] \tag{3}$$

is the overall system after equalization. The amount of intersymbol interference (ISI) defined as [1]

$$\text{ISI}\{g(n)\} = \frac{\sum_n |g(n)|^2 - \max\{|g(n)|^2, \forall n\}}{\max\{|g(n)|^2, \forall n\}} \tag{4}$$

has been used as a performance index of the designed $v[n]$. The smaller ISI implies the better performance.

A number of blind equalization algorithms using higher-order statistics (cumulants and moments) have been reported for designing $v[n]$ such as the well-known constant modulus algorithm (CMA) [2], inverse filter criteria (IFC) [3] and super-exponential algorithm (SEA) [1]. Chi, Feng and Chen [4] have reported the equivalence of IFC and SEA under certain conditions. In this paper, we further prove the equivalence of IFC, SEA and CMA under certain conditions, thus sharing some properties reported in [4–7] under these conditions. Furthermore, efficient implementations of these algorithms with preprocessing by linear prediction error (LPE) filter are presented including off-line processing and adaptive processing.

2. BACKGROUND

Let $\text{cum}\{x_1, ..., x_p\}$ denote the pth-order joint cumulant of random variables x_1, ..., x_p, and $\text{cum}\{e[n] : p, ...\} = \text{cum}\{x_1 = e[n], ..., x_p = e[n], ...\}$. For ease of later use, let us define the following notations

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$$
\begin{array}{rcl}
\mathbf{v} & = & (v[0], v[1], ..., v[L])^{\mathrm{T}} \\
\mathbf{x}[n] & = & (x[n], x[n-1], ..., x[n-L])^{\mathrm{T}} \\
f_k[n] & : & k\text{th}-\text{order forward prediction error} \\
b_k[n] & : & k\text{th}-\text{order backward prediction error} \\
\mathbf{f}_k[n] & = & (f_k[n], f_k[n-1], ..., f_k[n-L])^{\mathrm{T}} \\
\mathbf{b}[n] & = & (b_0[n], b_1[n], ..., b_L[n])^{\mathrm{T}} \\
C^u_{p,q} & = & \mathrm{cum}\{u[n]:p, u^*[n]:q\} \\
\mathrm{sgn}(\alpha) & : & \text{sign of real-valued } \alpha
\end{array}
$$

2.1. Lattice LPE Filter

The kth-order lattice LPE filter with reflection coefficients ρ_1, ρ_2, ..., ρ_k, simultaneously provides the forward prediction error $f_k[n]$ and backward prediction error $b_k[n]$, that can be expressed as follows:

$$f_k[n] = \sum_{i=0}^{k} a_k[i]x[n-i] \tag{5}$$

$$b_k[n] = \sum_{i=0}^{k} a_k^*[k-i]x[n-i] \tag{6}$$

where the superscript '*' denotes complex conjugation, $a_k[0] = 1$ and $a_k[1]$, $a_k[2]$, ..., $a_k[k]$, can be obtained from ρ_1, ρ_2, ..., ρ_k through the computationally efficient Levinson-Durbin recursion. Two facts regarding $f_k[n]$ and $b_k[n]$ are as follows [8]:

(F1) The kth-order LPE filter $a_k[i]$ is a whitening filter as k is sufficiently large, i.e.,

$$\mathbf{R}_{\mathbf{f}_k} = E\left[\mathbf{f}_k[n]\mathbf{f}_k^H[n]\right] \cong \sigma^2_{f,k}\mathbf{I} \tag{7}$$

for sufficiently large k where $\mathbf{I}$ is the $(L+1) \times (L+1)$ identity matrix.

(F2) $\mathbf{x}[n]$ and $\mathbf{b}[n]$ are causally invertible and

$$\mathbf{R}_{\mathbf{b}} = E\left[\mathbf{b}[n]\mathbf{b}^H[n]\right] = \mathrm{diag}(P_0, P_1, ..., P_L) \tag{8}$$

2.2. CMA

The CMA [2] finds the optimal equalizer $v[n]$ by minimizing the following cost function

$$J_{\mathrm{CM}}(\mathbf{v}) = E\left[(\gamma - |e[n]|^2)^2\right] \tag{9}$$

where $\gamma = E[|u[n]|^4]/E[|u[n]|^2]$. However, one has to resort to iterative optimization algorithms for searching the optimum $\mathbf{v}$.

2.3. SEA

Shalvi and Weinstein's SEA(p,q) [1] is an iterative algorithm that updates $\mathbf{v}$ by the following equations at each iteration:

$$\mathbf{v} = \mathbf{R}_{\mathbf{x}}^{-1}\mathbf{d}/||\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{d}|| \tag{10}$$

where $\mathbf{R}_{\mathbf{x}} = E[\mathbf{x}[n]\mathbf{x}^H[n]]$ and

$$\mathbf{d} = \mathrm{cum}\{e[n]:p, e^*[n]:q-1, \mathbf{x}^*[n]\}, \quad p+q \geq 3 \tag{11}$$

The SEA is a computationally efficient algorithm with fast convergence speed (in terms of ISI) but no guarantee of convergence for finite SNR and data.

2.4. IFC

The IFC(p,q) [3] find the optimum $\mathbf{v}$ by maximizing the following criteria:

$$J_{p,q}(\mathbf{v}) = \frac{\left|C^e_{p,q}\right|}{\left[C^e_{1,1}\right]^{(p+q)/2}}, \quad p+q \geq 3 \tag{12}$$

which is a highly nonlinear function of $v[n]$ without a closed-form solution for the optimum $\mathbf{v}$. Chi, Feng and Chen [4] proposed a fast gradient type iterative algorithm as follows:

***Algorithm 1*:**

At the ith iteration, $\mathbf{v}^{[i]}$ is obtained through the following two steps.

(T1) Update $\widehat{\mathbf{v}}$ using (10) with $e[n] = e^{[i-1]}[n]$ used in $\mathbf{d}$ (see (11)), and obtain the associated $e^{[i]}[n]$.

(T2) If $J_{p,q}(\widehat{\mathbf{v}}) > J_{p,q}(\mathbf{v}^{[i-1]})$, update $\mathbf{v}^{[i]} = \widehat{\mathbf{v}}$, otherwise update $\mathbf{v}^{[i]}$ by

$$\mathbf{v}^{[i]} = \mathbf{v}^{[i-1]} + \mu\ \mathrm{sgn}(C^u_{p,q})\widehat{\mathbf{v}} \tag{13}$$

such that $J_{p,q}(\mathbf{v}^{[i]}) > J_{p,q}(\mathbf{v}^{[i-1]})$, and obtain the associated $e^{[i]}[n]$.

Algorithm 1 requiring real $x[n]$, or complex $x[n]$ and $p = q$, shares the computational efficiency and convergence speed of the SEA with guaranteed convergence.

3. EQUIVALENCE OF SEA(2,2), IFC(2,2) AND CMA

Chi, Feng and Chen [4] have proven the following fact:

(F3) SEA(p,q) and IFC(p,q) are equivalent as $x[n]$ is real and $p+q \geq 3$ or as $x[n]$ is complex and $p = q \geq 2$.

As mentioned in (F2), $\mathbf{x}[n]$ and $\mathbf{b}[n]$ are causally invertible. Therefore, deconvolution with $\mathbf{x}[n]$ is equivalent to deconvolution with $\mathbf{b}[n]$. Let

$$e[n] = \mathbf{v}^{\mathrm{T}}\mathbf{b}[n] \tag{14}$$

Replacing $\mathbf{x}[n]$ and $\mathbf{R}_{\mathbf{x}}$ in (10) with $\mathbf{b}[n]$ and $\mathbf{R}_{\mathbf{b}}$, respectively, and replacing $e[n]$ in (11) with the one given by (14) for $p = q = 2$ through some simplification yields

$$\widehat{\mathbf{v}} = \mathbf{R}_{\mathbf{b}}^{-1}E\left[|e[n]|^2 e[n]\mathbf{b}^*[n]\right] \tag{15}$$

except for a scale factor. On the other hand, substituting (14) into $J_{\mathrm{CM}}(\mathbf{v})$ given by (9), one can easily show that the optimum $\widehat{\mathbf{v}}$ associated with the $J_{\mathrm{CM}}(\mathbf{v})$ is the same as the one given by (15) except for a scale factor. Therefore, we have shown the following theorem:

Theorem 1. Both SEA(p,q) with $p=q=2$ and CMA are equivalent.

By (F3) and Theorem 1, we have the following fact:

(F4) The CMA, IFC(p,q) and SEA(p,q) are equivalent as $p=q=2$. Therefore, they share some properties reported in [4–7], such as perfect equalization property and relation to nonblind minimum mean square error (MMSE) equalizer.

4. LATTICE IMPLEMENTATIONS

Let us present lattice implementations for SEA(p,q), IFC(p,q) and CMA only for the case of $p=q=2$ below.

4.1. Off-Line Processing

Feng and Chi have reported two off-line lattice SEA (LSEA) [9] using $\mathbf{b}_k[n]$ and $\mathbf{f}_k[n]$, respectively. Next, let us present two lattice implementations for IFC that are modifications of Algorithm 1 with $\mathbf{x}[n]$ replaced by $\mathbf{b}_k[n]$ and $\mathbf{f}_k[n]$, respectively.

LIFC-B Algorithm: At the ith iteration, $\mathbf{v}^{[i]}$ is obtained through the following two steps.

(S1) Compute $\widehat{\mathbf{v}}$ by (15) where $e[n]=e^{[i-1]}[n]$ is obtained by (14) at the $(i-1)$th iteration.

(S2) If $J_{2,2}(\widehat{\mathbf{v}}) > J_{2,2}(\mathbf{v}^{[i-1]})$, update $\mathbf{v}^{[i]}=\widehat{\mathbf{v}}$, otherwise update $\mathbf{v}[i]$ through a gradient-type optimization procedure with the gradient

$$\nabla J_{2,2} \propto \mathrm{sgn}(C_{2,2}^{u})\mathbf{R}_{\mathbf{b}}(\widehat{\mathbf{v}}-\mathbf{v}^{[i-1]}). \qquad (16)$$

LIFC-F Algorithm: Let

$$e[n]=\mathbf{v}^{\mathrm{T}}\mathbf{f}_k[n] \qquad (17)$$

where k is sufficiently large such that (F1) applies to $\mathbf{f}_k[n]$. At the ith iteration, $\mathbf{v}^{[i]}$ is obtained through the same procedure as the previous LIFC-B algorithm except that $\mathbf{b}[n]$ and $\mathbf{R}_{\mathbf{b}}$ are replaced by $\mathbf{f}_k[n]$ and $\mathbf{R}_{\mathbf{f}_k}$, respectively, with $e^{[i]}[n]$ obtained by (17) and $\nabla J_{2,2}$ obtained by

$$\nabla J_{2,2} \propto \mathrm{sgn}(C_{2,2}^{u})(\widehat{\mathbf{v}}-\mathbf{v}^{[i-1]}). \qquad (18)$$

A worthy remark regarding the proposed LIFC-B and LIFC-F algorithms is as follows:

(R1) The proposed LIFC-B and LIFC-F algorithms are computationally efficient (without need of matrix inversion) with guaranteed convergence, whereas the latter converges faster than the former since $f_k[n]$ approximates an amplitude equalized signal by (F1).

(R2) As deriving the LIFC-B and LIFC-F algorithms (maximizing $J_{2,2}$), one can readily obtain two lattice CMA algorithms (minimizing J_{CM}), using $\mathbf{b}_k[n]$ and $\mathbf{f}_k[n]$, respectively, that also share the implementation merits of the LIFC-B and LIFC-F algorithms mentioned in (R1).

4.2. Adaptive Processing

Let $\mathbf{v}_n$ denote the estimate of $\mathbf{v}$ as $\mathbf{x}[n]$ is processed. An adaptive SEA reported in [1] is as follows:

$$\mathbf{v}_{n+1}=\mathbf{v}_n+\mu\mathbf{Q}_{n+1}\mathbf{x}^*[n+1]e[n](\gamma-|e[n]|^2) \qquad (19)$$

$$\mathbf{v}_{n+1}=\mathbf{v}_{n+1}/\|\mathbf{v}_{n+1}\| \qquad (20)$$

where μ is the step size parameter, and

$$e[n+1]=\mathbf{v}_{n+1}^{\mathrm{T}}\mathbf{x}[n+1] \qquad (21)$$

$$\mathbf{Q}_{n+1}^{-1}=(1-\mu)\mathbf{Q}_n^{-1}+\mu\mathbf{x}[n+1]\mathbf{x}^H[n+1] \qquad (22)$$

With $\mathbf{Q}_{n+1}$ and $\mathbf{x}[n]$ in (19) replaced by $\mathbf{R}_{\mathbf{f}_k}^{-1}$ and $\mathbf{f}_k[n]$, one can obtain

$$\begin{aligned} \mathbf{v}_{n+1} &= \mathbf{v}_n+\mu\{\gamma-|e[n]|^2\}e[n]\mathbf{f}_k^*[n+1] && (23)\\ e[n+1] &= \mathbf{v}_{n+1}^{\mathrm{T}}\mathbf{f}_k[n+1] && (24)\end{aligned}$$

Lattice SE-IF-CM Algorithm: For each $x[n+1]$, two signal processing steps are performed as follows:

(U1) Obtain $f_k[n+1]$ by processing $x[n+1]$ with the adaptive least squares lattice (LSL) LPE filter [8].

(U2) Update $\mathbf{v}_{n+1}$ and $e[n+1]$ using (23) and (24), respectively.

Two worthy remarks regarding the lattice SE-IF-CM algorithm are as follows:

(R3) The lattice SE-IF-CM algorithm is exactly a lattice CMA algorithm since (U2) is the same as the adaptive CMA [2] and an adaptive IFC algorithm [3] except that $\mathbf{f}_k[n]$ is replaced with $\mathbf{x}[n]$.

(R4) The proposed lattice SE-IF-CM algorithm with low computational load (without matrix multiplication operations) converges faster than the adaptive SEA given by (19) through (22) and the adaptive CMA since the adaptive LSL algorithm in (U1) performs as a fast amplitude equalizer.

5. SIMULATION RESULTS

Two examples are presented to support our analytic results and the lattice structure based algorithms.

Example 1: Off-line Processing

The source signal $u[n]$ was assumed to be a 4-QAM signal with unity variance and a real channel $h[n]$ was taken from [1] as plotted in Figure 1(a). The equalizer $v[n]$ was assumed to be a causal FIR filter of order $L = 50$. Thirty independent runs for data length $N = 4096$ and SNR = 20 dB (complex white Gaussian noise) were performed using CMA and SEA(2,2) with the initial condition $v[n] = \delta[n-L/2]$, respectively. The averages of thirty independent estimates of equalizer $v[n]$ obtained using CMA and SEA(2,2) are displayed in Figures 1(b) and 1(c), respectively, where only equalizer real parts are shown since imaginary parts are almost zero. These results justify Theorem 1.

Moreover, Algorithm 1, LIFC-B and LIFC-F algorithms and a gradient-based IFC algorithm were also employed to process the same simulation data. Figure 2 shows the average of the thirty $J_{2,2}$'s with respect to iteration number associated with LIFC-F (k=50) algorithm (dash line), LIFC-B algorithm (dash-dotted line), Algorithm 1 (dotted line) and the gradient-based IFC algorithm (solid line). Figure 2 depicts that the LIFC-F algorithm and Algorithm 1 converge faster than the other two algorithms (see (R1)) and the gradient-based IFC algorithm converges slower than all the other algorithms. These simulation results support the efficacy of the proposed LIFC-B and LIFC-F algorithms.

Example 2: Adaptive Processing

The source signal $u[n]$ was assumed to be a 2-PAM $(+1, -1)$ signal. The same channel $h[n]$ as shown in Figure 1(a) was used, and SNR = 20 dB (real white Gaussian noise). Figure 3 shows some simulation results (average of thirty independent ISI's versus iteration number) for $L = 24$ using the adaptive SEA with $p = q = 2$ and $\mu = 0.0026$, the adaptive CMA with $\mu = 0.00215$ and the proposed adaptive lattice SE-IF-CM algorithm with $k = 24$ and $\mu = 0.002$. Note that the value of the step size μ used by each adaptive algorithm was chosen through some trial-and-errors such that its performance is "best" in terms of convergence speed and ISI. One can see, from Figure 3, that the proposed adaptive lattice SE-IF-CM algorithm (solid line) converges faster than the other two adaptive algorithms with ISI slightly smaller than those associated with the other two adaptive algorithms. These simulation results justify the efficacy of the proposed adaptive lattice SE-IF-CM algorithm (see (R4)).

6. CONCLUSIONS

We have shown the equivalence of the CMA, SEA(p,q) and IFC(p,q) for $p = q = 2$ as presented in Theorem 1 and (F4), and therefore, any performance analyses for one of them apply to the others. Furthermore, two computationally efficient off-line processing algorithms, LIFC-F and LIFC-B algorithms for $p = q = 2$ were presented, while the former is preferable to both the latter and Chi, Feng and Chen's Algorithm 1 due to faster convergence (see (R1)). For adaptive processing, a computationally efficient lattice SE-IF-CM algorithm for $p = q = 2$ was presented that has computational complexity similar to the adaptive CMA and converges faster than both the adaptive SEA and the adaptive CMA with similar resultant ISI (see (R3) and (R4)). The efficacy of the proposed adaptive lattice SE-IF-CM algorithm and the proposed analytic results were supported by some simulation results. As a final remark, for $p \neq q$ or $p = q \neq 2$, lattice implementations of the SEA and IFC can be similarly developed.

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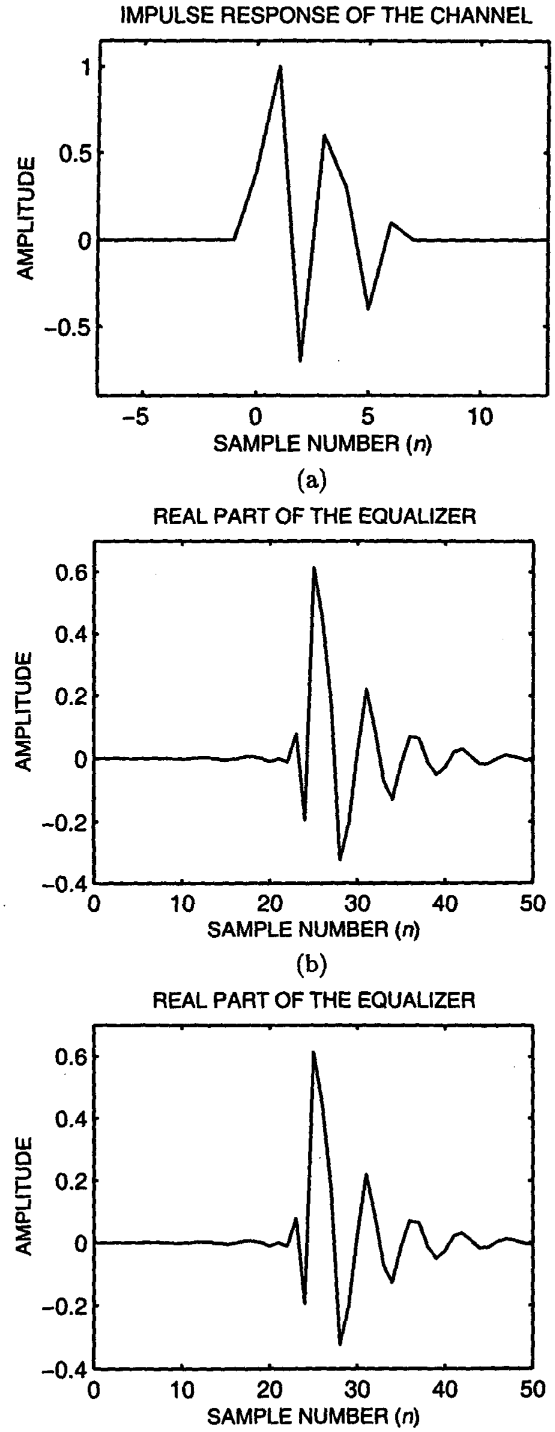

(c)

Figure 1. Simulation results for $N = 4096$ and SNR= 20 dB. (a) The channel impulse response; (b) average of thirty estimates of equalizer $v[n]$ associated with CMA and (c) that associated with SEA(2,2).

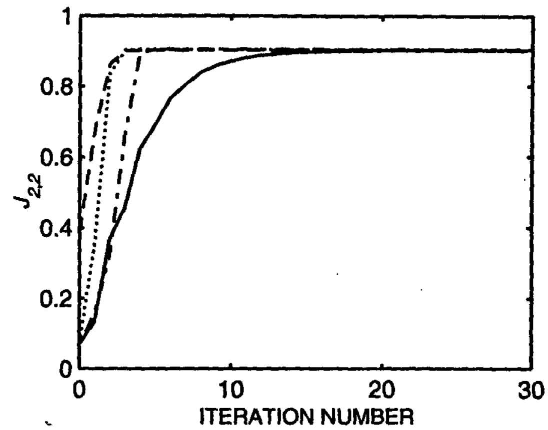

Figure 2. Average of thirty $J_{2,2}$'s associated with LIFC-F (k=50) (dash line) algorithm, LIFC-B algorithm (dash-dotted line), Algorithm 1 (dotted line) and the gradient-based IFC algorithm (solid line).

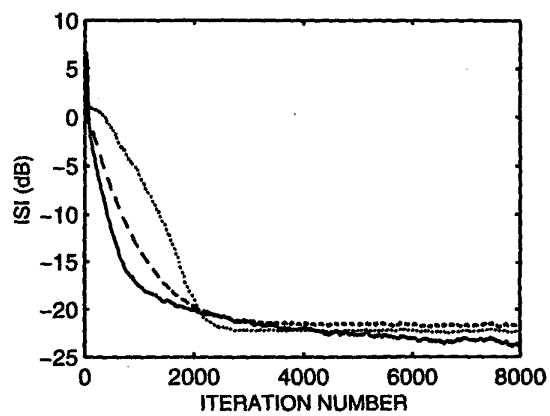

Figure 3. Simulation results (ISI versus iteration number) using the adaptive CMA (dash line) with $\mu = 0.00215$, the adaptive SEA (dotted line) with $p = q = 2$ and $\mu = 0.0026$ and the proposed adaptive lattice SE-IF-CM algorithm (solid line) with $k = 24$ and $\mu = 0.002$, respectively.